

# Retro-analytical Reasoning IQ tests for the High Range.

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## Abstract

In this paper we provide a proposal to produce a new, computer-based generation of IQ tests for the high range (from  $+3\sigma$  and higher above the mean) without being prone to cheating.

IQ tests are at a considerable risk of this kind, and episodes of cheating have actually occurred, even on the most famous standardized and supervised IQ tests.

As soon as a test is administered to a candidate, there is no longer the certainty that its items will remain secret; this is as a consequence of the “static nature” of IQ tests, and the problem may be solved through a dynamic system based on different items for each submission of the test.

Our proposal is based on a method to construct new integer sequences starting from a given and explicit set of sequences, using *information asymmetry* (i.e. different items for each session). The related solving of problems will be linked to inference and retro-analytical reasoning, similar to the retrograde analysis of chess problems.

## Keywords

IQ test, retro-analytics reasoning, World Intelligence Network, gifted, giftedness screening, derivation process, standardized tests, cheating, integer sequences, Automatic Sequences Generator, High Range Test, Italy.

# 1 Introduction and problem statement: IQ 200+ ( $\sigma=24$ ) for the average candidate

Most of the standardized and supervised IQ tests are not without risk if we want to use them for giftedness screening (even if they represent the best choice for an average person's reasoning skills evaluation, or IQ deficit diagnosis).

It is regrettable that certain people try to sell (usually on the Web) some excellent standardized supervised tests, as evidenced, for example, by the following screenshots (taken from eBay):

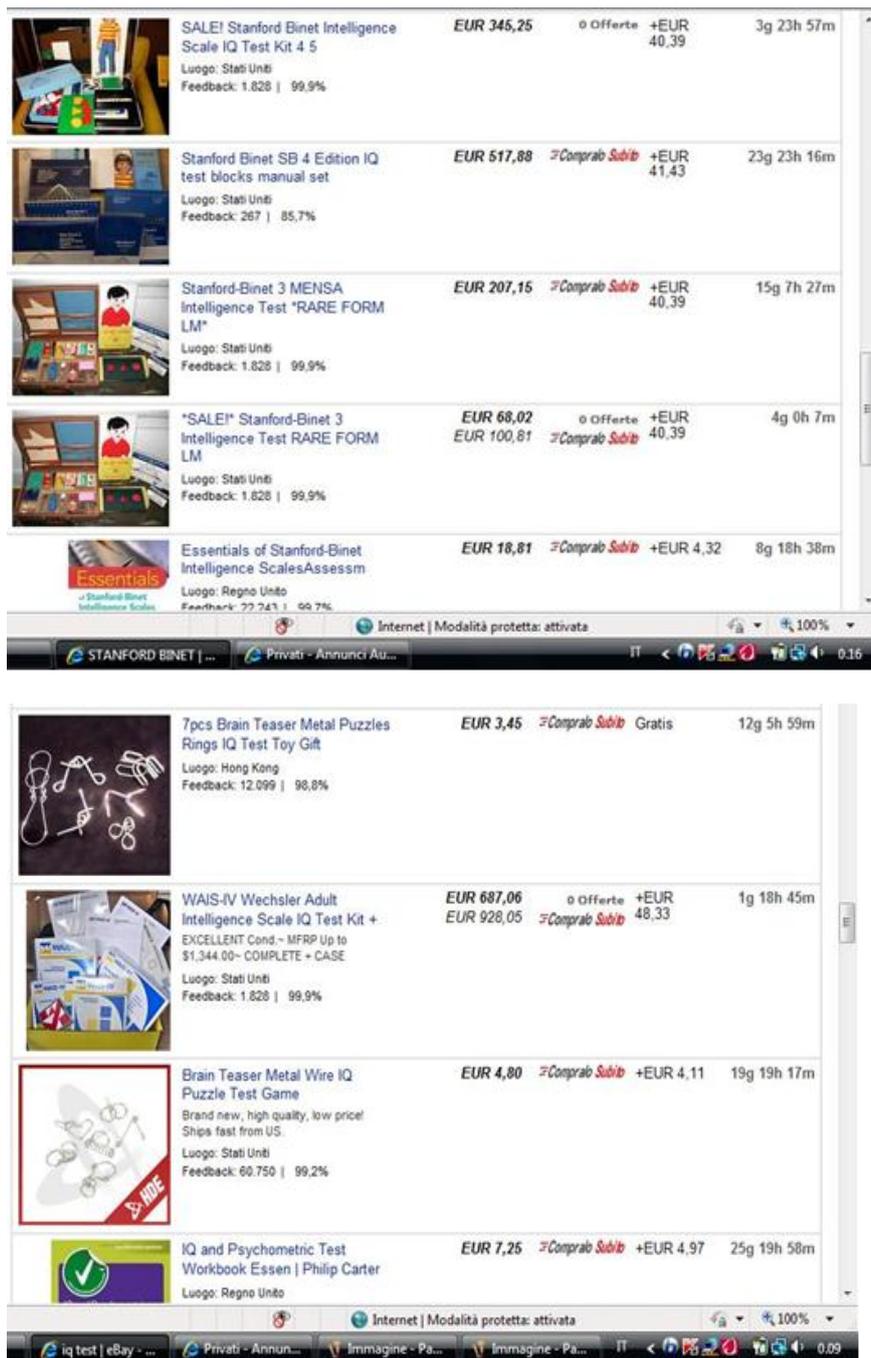


Figure 1: The result of a professional search of IQ tests on eBay using the names of some famous standardized and supervised tests as keywords.

If someone succeeded in buying some tests of this kind, it would be easy to cheat on them, achieving a perfect score under the supervision of a serious psychologist, in front of the Media too. A couple of years ago, a displeasing episode occurred featuring the Cattell Culture Fair III (form A+B) done by a candidate who got a perfect score under the television eye, but who was unable to reach a 130 ( $\sigma=15$ ) performance on a similar test, i.e. Raven's Advanced Progressive Matrices with a time limit of 60 minutes [11].

In order to reduce this cheating risk, one possible solution is to adopt qualitative [8] IQ tests, even if episodes of cheating have also occurred on them (for example, the Get-y test attack in 2010, performed by a group of Chinese [<http://www.epiqsociety.net/get/>]).

As far as unsupervised IQ tests for the high range are concerned, there are many examples of "excellent works" which, unfortunately, are no longer accepted for admission to high IQ societies [2].

The following text is taken from the official membership page of the "Prometheus Society" [<http://216.224.180.96/~prom/oldsite/membership/index.html>]:

***"The Prometheus Society officers have voted that Mega27 score sheets dated after 11/27/99 will not be accepted for admission to the Prometheus Society. This measure is being taken in response to recent compromises of the Mega27 test."***

The following text comes from the official membership page of the "One-in-a-Thousand Society" (OATH) [<http://www.oathsociety.com/membership.html>]:

***SOME QUALIFYING SCORES:***

.....

- ***Mega Test (before 1995) 24 right***
- ***Titan Test 24 right (not taken after 2/27/2011)***
- ***Langdon Adult Intelligence Test (before 1994) 150 IQ***

.....

Besides, on the "Artistic Minds Society" [<http://www.materials-synthesis.com/amc/>], you can read:

***"The CFNSE test has unfortunately been compromised and it is no longer being scored. Older CFSNE scores though will still be accepted."***

Several other examples of IQ test scores, which are accepted in most high IQ societies "only if got before certain dates", can be found quite easily, simply by looking at the *membership pages* of these groups.

The most common reason why some unsupervised high range IQ tests are no longer accepted is related to their impairment, often due to the publication of the solution to some items on the Web, or within other public contexts.

Of course, great tests, like the ones previously mentioned, are still useful (if still scored by the authors) for auto-evaluation purposes. Nevertheless, each time a test is no longer accepted for admission into certain high IQ societies, inevitably, it loses part of its interest.

With reference to what we have explained above, it would be optimal to discover at least one family of supervised IQ tests for the high range which is immune from the risk of cheating. It is important to point out that the risk we are talking about involves, almost exclusively, tests used with the aim of investigating high performances.

A new proposal to create a new kind of totally culture free numerical high IQ test is based on a method to construct new integer sequences starting from a given and explicit set of sequences. The related solving of problems will be linked to inference and retro-analytics reasoning, similar to the retrograde analysis of chess problems [7-9]. Our idea is to provide some software which is able to produce a huge amount of “varying difficulty” items, starting from a short and public input. This method is quite similar to the RSA encryption algorithm and is based on asymmetric information. The candidates have to provide an articulated answer for each item and this approach cuts out for the most part the “false positive results”.

## 2.1 The derivation sequences method to create a wide set of supervised high range IQ tests

The method we are going to describe assures us about the possibility of creating a wide set of distinct IQ tests, with an equivalent *raw scores*  $\rightarrow$  *IQ scores* distribution. This means that, when the norming process has been completed, the raw score obtained in any given test belonging to this family will be associated with only one IQ score, regardless of the specific test taken. Passing from one test to another one, the raw score  $\rightarrow$  IQ score conversion table will remain unchanged: this will provide us with the possibility of using a given set of items “in only one session”, which is, as we explained above, the main target of this work.

For the norming purpose, we can use one (random) specific test, or a small set of tests. We need to calculate different conversion tables (which implies the same number of norms) if we want to test children below 17 years of age; furthermore, if we are interested in IQ tests of various time durations, or if we are focusing on different IQs ranges, the nearer the result is to the middle of the range we are considering, the smaller the relative statistical error will be [3].

## 2.2 Main idea

Our method is based on a process to derive new sets of sequences starting from a small amount of fixed sequences (9 to 12). The underlying rules are public. They are included in the *incipit* of the test administered by the psychologist/psychometrist.

In order to give the candidates the wherewithal for understanding how each item is constructed, we need to point out to them the mathematical idea which is at the bottom of the “derivation process” we have adopted.

The Description of the “derivation process.”

The derivation process is the process we use to create an item of the final IQ test, starting from a given set of integer sequences. The whole process consists of one or more “steps of derivation” (that is, some items are created through a single step of derivation, other items through more than one step).

Each step involves two sequences:

- ✓ The first one (from here on, “FS”) represents a “sequence of values”, because some of its elements are selected (according to a certain criterion which we are going to describe) and used as “values of the elements which constitute the output sequence” (from here on, “OS”), which is the sequence produced by the current step of derivation.  
The FS is included in the initial set of integer sequences for the first step of derivation (in this case, we call the FS the “mother sequence”); for steps of derivation whose order is higher than one, the FS is the OS of the previous step.
- ✓ The second sequence (from here on, “SS”) represents a “sequence of positions”, because the various values of its elements indicate “the positions occupied by the elements of the FS which will be inserted into the OS”.

During a given step of derivation, we run through the SS. For each element of the SS, we insert into the OS an element belonging to the FS: the element chosen from the FS is the one which occupies (within the FS itself) a position equal to the value of the current element of the SS. According to this methodology, the SS represents the “*rule*” that is applied to the FS, in order to identify which elements of the latter will be inserted into the OS of the current step of derivation.

The following table shows an example of the derivation process; both the mother sequence and the SS constitute the canonical prime sequence (OEIS [A000040](http://oeis.org/A000040) - <http://oeis.org/A000040>):

Zero order primes, p<100	First order primes, p<100	Second order primes, p<100	Third order primes, p<100	Fourth order primes, p<100
2	///			
3	3	///		
5	5	5	///	

7				
11	11	11	11	///
13				
17	17			
19				
23				
29				
31	31	31	31	<b>31</b>
37				
41	41			
43				
47				
53				
59	59	59		
61				
67	67			
71				
73				
79				
83	83			
89				
97				

Table 1: The derivation process considering as unique rule of derivation the *Prime numbers* < 100 subset.

Using the same principle above, we can define the set of the derived sequences which are constructed from the rule “the cube of a conventional prime”. The smallest of them is 19, the eighth conventional prime: in fact,  $8=2^3$ .

What has been shown for the *positional primes* is valid for any integer sequence: it is just sufficient to apply the same criterion iteratively.

For example (putting between brackets the progressive positional numbers referring to the next term of the sequence):

Fibonacci sequence  $\rightarrow$  zero order (mother sequence)  $\leftrightarrow$

$\leftrightarrow (1)1,(2)1,(3)2,(4)3,(5)5,(6)8,(7)13,(8)21,(9)34,(10)55,(11)89,(12)144,(13)233,\dots$

Order 1 sequence  $\leftrightarrow (1)1,(2)1,(3)1,(4)2,(5)5,(6)21,(7)233,(8)10946,(9)5702887,\dots$

Order 2 sequence  $\leftrightarrow (1)1,(2)1,(3)1,(4)1,(5)5,\dots$

...

Order 4 sequence  $n \geq 2 \leftrightarrow (1)1,(2)1,(3)1,(4)1,(5)5,\dots$

**Observation.** In this case, when the order of the derived sequence is  $\geq 2$ , the first 5 terms are fixed: (1)1,(2)1,(3)1,(4)1,(5)5 (for further investigations, see the appendix below).

Combining multiple rules (related to different mother sequences), we can create an unlimited number of order  $\geq 1$  subsequences.

For example:

Mother sequence (zero order): Fibonacci

[(1)1,(2)1,(3)2,(4)3,(5)5,(6)8,(7)13,(8)21,(9)34,(10)55,(11)89,(12)144,(13)233,(14)377,(15)610,  
(16)987,...]

Order 1 sequence: Fibonacci

[(1)1,(2)1,(3)2,(4)3,(5)5,(6)8,(7)13,(8)21,(9)34,(10)55,(11)89,(12)144,(13)233,(14)377,(15)610,  
(16)987,...]

Order 2 sequence: Prime numbers

[(1)2,(2)3,(3)5,(4)7,(5)11,(6)13,(7)17,(8)19,(9)23,(10)29,(11)31,(12)37,(13)41,(14)43,(15)47,  
(16)53,...]

Order 3 sequence: Perfect squares

[(1)1,(2)4,(3)9,(4)16,(5)25,(6)36,(7)49,(8)64,(9)81,(10)100,(11)121,(12)144,...]

Evolution:

Order 1 sequence (Fibonacci  $\rightarrow$  Fibonacci)  $\leftrightarrow$

$\leftrightarrow$  (1)1,(2)1,(3)1,(4)2,(5)5,(6)21,(7)233,(8)10946,(9)5702887,(10)139583862445,...

Order 2 sequence (Fibonacci  $\rightarrow$  Fibonacci  $\rightarrow$  Prime numbers)  $\leftrightarrow$  (1)1,(2)1,(3)5,(4)233,...

Order 3 sequence (Fibonacci  $\rightarrow$  Fibonacci  $\rightarrow$  Prime numbers  $\rightarrow$  Perfect squares)  $\leftrightarrow$  (1)1,(2)233,...

**Observation.** As already described above, it is self-evident that the second and the thirteenth element (say  $a_2$  and  $a_{13}$ ) from the “mother sequence” also belong to the “order 3 sequence” and, for the same reason, they fit all the order  $\leq 3$  sequences as well, according to the established hierarchical structure.

Starting from a given sequence, it is possible to create infinite others (also coincident), but the condition (necessary, but not sufficient) postulates that the set of the terms that compose it has infinite cardinality (that is, the number of elements of the mother sequence must be infinite).

We have to distinguish between two kinds of sequences:

- ✓ incremental sequences: sequences such that  $a_n < a_{(n+1)}$ , where  $a_n$  indicates the n-th term of the sequence;
- ✓ non incremental sequences: sequences which do not respect the previous property.

In the second case, proceeding with the derived sequences (order greater than 1), we could obtain an endless chain of sequences which, starting from a certain step, repeat themselves with a precise regularity (infinitely). This happens when the original sequence admits an upper bound. Anyway

this is a necessary, but not sufficient, condition (just think of an odd sequence whose terms are always equal to 3, while the even elements are constituted by powers of 2, arranged in ascending order: starting from the second step, the sequence will admit **3** as a majorant). Let us think, as “borderline cases”, of the sequences derived from the decimal expansions of “**e**”, “**pi**” and “ $\sqrt{2}$ ” (see Appendix), whose terms are strictly less than 10 (in these cases, you can choose  $m \geq 10$  as a majorant).

**N.B.**

For practical use, we denote by “IS” the mother sequences used by the algorithm (derivation process) and “OS” for the final outputs (the sequences obtained considering the amount of “steps” we want).

### 3 Concrete rules for the implementation of the test

These are the operating rules to construct one IQ test item:

- 1- Maximum amount of derivations: 3 steps (which means 3 underlying rules).
- 2- Not more than 15% of items based on 4 different rules.
- 3- Globally, each test should contain between 25 and 40 items.
- 4- Every term must be smaller than  $10^{10}$  (from 1 to 9 digit long numbers).
- 5- Each IS should contain a minimum of 500 terms, while the OS must contain from 7 up to a maximum of 20 elements (the text will show the minimum quantity of terms which is sufficient to get a unique solution to each given item – see rule 10).
- 6- Items have to be randomly shuffled inside the test (with no hints about the difficulty of the item itself, and without items sorted by difficulty).
- 7- The amount of items based on a given number of derivations (for example, 3 with only one derivation, 20 with 2 derivations, 12 with 3 derivations) must be public and will be included inside the pamphlet delivered to each candidate during the preliminary briefing (see section 6).
- 8- The text (public sequences set) has to contain between 50% and 65% of strictly **non decremental sequences** (each term must be smaller than the next one) and from 35% to 50% of **free sequences** (which are not subjected to any constraints). The standard ratio would be 60% of strictly non decremental sequences and 40% of free sequences.
- 9- From 9 up to 12 of only public sequences (the standard amount would be 11).
- 10- Each item should contain the **minimum amount** of terms which propagate a unique solution (only one possible step-to-step pattern being involved), provided that the amount of the terms belonging to the item itself is at least 7. Otherwise, the item has to list the first 7 terms of the final sequence.

11- The psychologist/psychometrist who administers the test should take from 30 to 45 minutes to properly explain the test rules to the candidates. At the beginning of the informative briefing, he must deliver the full equipment package to each candidate.

12a Whenever one (or more) of the previous parameters is different from a given test to another one, different norms must be used for the scoring.

12b (Reminder)- Last but not least, each blank term (indicated by a question mark) must be the result of only one retro-analytical reasoning pattern.

An example of an IQ test for the high range, probably suitable for candidates with an IQ at or above the  $3\sigma$  cut-off [4] (e.g. students who constitute a valuable reservoir from which prestigious universities around the world can draw resources), might be composed of a total of 35 (shuffled) items, broken down as follows:

$$\left\{ \begin{array}{l} 3 \text{ items: 1 derivation;} \\ 20 \text{ items: 2 derivations (5 using repeated rules, plus 15 only based on different rules);} \\ 12 \text{ items: 3 derivations (7 using repeated rules, plus 5 only based on different rules).} \end{array} \right.$$

An appropriate time limit would be around 140-180 minutes.

It is important to point out that, considering “n” public sequences (IS), and “k” derivation steps, we obtain  $n^{(k+1)}$  total sequences. For example, if  $n=11$  and  $k=3$ , we get 14641 possible sequences (OS), even if some of them will not be useful for our purpose (to create valid items for the test). Despite this being a little inconvenient, a large amount of sequences for a lot of different tests (based on the same set of IS) will still be available.

Under the previous assumption, we have:

1 step  $\rightarrow$  121 sequences (at most);

2 steps  $\rightarrow$  1331 sequences (at most);

3 steps  $\rightarrow$  14641 sequences (at most).

Starting from the given set of IS, the software produces one printout for each derivation step used (i.e. a 3 steps OS creates 3 distinct printouts of different magnitude). The IS set (the input) is changed automatically by the software itself, until the generated printout contains (at least) 6 distinct tests (whereas, under the assumptions above, the maximum quantity of different achievable tests is equal – using the “floor operator” – to  $\left\lfloor \frac{n^2}{3} \right\rfloor$ , where “n” represents the numerosness of the IS considered by the test).

**N.B.**

It is not possible that the software will be unable to produce a minimum of 6 tests with no common “useful” items amongst them.

In practice (under the constraints we have stated), a possible scenario (for one of the five most difficult items of the test) could be as follows:

### Public set of incremental sequences:

- 1) *Golomb's numbers*: (1)1, (2)2, (3)2, (4)3, (5)3, (6)4, (7)4, (8)4, (9)5, (10)5, (11)5, (12)6, (13)6, (14)6, (15)6, (16)7, (17)7, (18)7, (19)7, (20)8, (21)8, (22)8, (23)8, (24)9, (25)9, (26)9, (27)9, (28)9, (29)10, (30)10, (31)10, (32)10, (33)10, (34)11, (35)11, (36)11, (37)11, (38)11, (39)12, ...
- 2) *Happy numbers [below 500]*: (1)1, (2)7, (3)10, (4)13, (5)19, (6)23, (7)28, (8)31, (9)32, (10)44, (11)49, (12)68, (13)70, (14)79, (15)82, (16)86, (17)91, (18)94, (19)97, (20)100, (21)103, (22)109, (23)129, (24)130, (25)133, (26)139, (27)167, (28)176, (29)188, (30)190, (31)192, (32)193, ...
- 3) *Padovan numbers [+1]*: (1)2, (2)1, (3)1, (4)2, (5)1, (6)2, (7)2, (8)2, (9)3, (10)3, (11)4, (12)5, (13)6, (14)8, (15)10, (16)13, (17)17, (18)22, (19)29, (20)38, (21)50, (22)66, (23)87, (24)115, (25)152, (26)201, (27)266, (28)352, (29)466, (30)617, (31)817, (32)1082, (33)1433, (34)1898, ...
- 4) *Practical numbers*: (1)1, (2)2, (3)4, (4)6, (5)8, (6)12, (7)16, (8)18, (9)20, (10)24, (11)28, (12)30, (13)32, (14)36, (15)40, (16)42, (17)48, (18)54, (19)56, (20)60, (21)64, (22)66, (23)72, (24)78, (25)80, (26)84, (27)88, (28)90, (29)96, (30)100, (31)104, (32)108, (33)112, (34)120, (35)126, ...
- 5) *Partition numbers*: (1)1, (2)1, (3)2, (4)3, (5)5, (6)7, (7)11, (8)15, (9)22, (10)30, (11)42, (12)56, (13)77, (14)101, (15)135, (16)176, (17)231, (18)297, (19)385, (20)490, (21)627, (22)792, (23)1002, (24)1255, (25)1575, (26)1958, (27)2436, (28)3010, (29)3718, (30)4565, (31)5604, (32)6842, ...
- 6) *Ulam numbers*: (1)1, (2)2, (3)3, (4)4, (5)6, (6)8, (7)11, (8)13, (9)16, (10)18, (11)26, (12)28, (13)36, (14)38, (15)47, (16)48, (17)53, (18)57, (19)62, (20)69, (21)72, (22)77, (23)82, (24)87, (25)97, (26)99, (27)102, (28)106, (29)114, (30)126, (31)131, (32)138, (33)145, (34)148, (35)155, (36)175, ...

### N.B.

In this case, we have chosen sequences from [[http://en.wikipedia.org/wiki/Integer\\_sequence](http://en.wikipedia.org/wiki/Integer_sequence)] but we can also consider  $a_{(n)} = 7 * k + 3$  (where  $k = 0, 1, 2, 3, \dots$ ), or  $a_{(n)} = a_{(n-1)} + n$  (where the first term is equal to, let us say, 4), etc.

### Public set of free sequences:

- 7) *Kalaskoski sequence*: (1)1, (2)2, (3)2, (4)1, (5)1, (6)2, (7)1, (8)2, (9)2, (10)1, (11)2, (12)2, (13)1, (14)1, (15)2, (16)1, (17)1, (18)2, (19)2, (20)1, (21)2, (22)1, (23)1, (24)2, (25)1, (26)2, (27)2, (28)1, (29)1, (30)2, (31)1, (32)1, (33)2, (34)1, (35)2, (36)2, (37)1, (38)2, (39)2, (40)1, ...
- 8) *Digital sum of n (n ≥ 1)*: (1)1, (2)2, (3)3, (4)4, (5)5, (6)6, (7)7, (8)8, (9)9, (10)1, (11)2, (12)3, (13)4, (14)5, (15)6, (16)7, (17)8, (18)9, (19)10, (20)2, (21)3, (22)4, (23)5, (24)6, (25)7, (26)8, (27)9, (28)10, (29)11, (30)3, (31)4, (32)5, (33)6, (34)7, (35)8, (36)9, (37)10, (38)11, (39)12, (40)4, (41)5, ...
- 9) *Decimal expansion of e (deleting the terms equal to zero)*: (1)2, (2)7, (3)1, (4)8, (5)2, (6)8, (7)1, (8)8, (9)2, (10)8, (11)4, (12)5, (13)9, (14)4, (15)5, (16)2, (17)3, (18)5, (19)3, (20)6, (21)2, (22)8, (23)7, (24)4, (25)7, (26)1, (27)3, (28)5, (29)2, (30)6, (31)6, (32)2, (33)4, (34)9, (35)7, (36)7, (37)5, ...
- 10) *Baum – Sweet sequence [+2]*: (1)3, (2)3, (3)2, (4)3, (5)3, (6)2, (7)2, (8)3, (9)2, (10)3, (11)2, (12)2, (13)3, (14)2, (15)2, (16)3, (17)3, (18)2, (19)2, (20)3, (21)2, (22)2, (23)2, (24)2, (25)2, (26)3, (27)2, (28)2, (29)3, (30)2, (31)2, (32)3, (33)2, (34)3, (35)2, (36)2, (37)3, (38)2, (39)2, (40)3, ...

Thus, a possible item can be: **2, 11, 1, ???, 13, 13, 1, ...**

The only acceptable solution is “**13 ; [6,9,1,4]**”, in fact, we have chosen the *Ulam numbers* as IS (the mother sequence), the *Decimal expansion of “e” (deleting the terms equal to zeros)* rule for the first step, the *Golomb’s numbers* rule for the second one and, at the end, the *Practical numbers* rule for the third step of derivation.

## 4.1 General description of the software (by G. Morelli & M. Ripà): how does it work?

ASG (Automatic Sequences Generator) is software created in order to automate the generation process of the IQ test items.

The input of the program consists of a set of 9, 10, 11 or 12 input sequences (“IS”).

For reasons of simplicity, from here on we fix the amount of the IS we consider at 11.

The IS are provided through an ASCII sequential file.

In order to identify each single sequence, the set of IS is sorted according to the position (from 1 to 11) of each sequence within the input file.

The output of the program consists of an ASCII sequential file, which contains any sequence that can be obtained through 1, 2 or 3 “steps of derivation” (see below for a detailed explanation) applied to the IS included in the input file.

In each output sequence, each element is preceded by a number in brackets, which indicates the position of the element within the OS.

The output file is sorted in alphabetical order (according to the ASCII code of each byte). Thanks to this kind of disposition, a generic OS which occupies the position number N can be quickly compared to the ones in the positions number N-1 and N+1: this lets us immediately identify which items can actually be used in the test without any risk of having more than one possible solution.

### **N.B.**

Each OS is “self-describing”. This means that each OS contains (at the end of the sequence) from 1 to 4 reserved numerical fields, which thus constitutes the “unique code of the sequence”.

This code describes the “story” of a sequence, identifying, step by step, the whole derivation process used for creating that sequence. Each filing of the code is a number between 1 and 11, because it represents the position of an IS within the set of IS (which is sorted, as previously said).

In details:

- the first field indicates the “mother sequence”;
- the second field indicates the IS used as “derivation rule” during the first step of derivation;
- the third field (if present) indicates the IS used as “derivation rule” during the second step of derivation;
- the fourth field (if present) indicates the IS used as “derivation rule” during the third step of derivation.

**Observation.** Of course, the fields of the code of each OS are not taken into account during the sort of the output file.

## **4.2 An output example**

The following text shows three consecutive output sequences, randomly selected from the OS created by the software (in this context, also listing the whole set of IS is not relevant):

.....  
 .....  
 .....

Code: <9,1,2,8>

Sequence: (1)3,(2)9,(3)11,(4)16,(5)18,(6)26,(7)41,(8)201 .....

Code: <9,8,2,8>

Sequence: (1)3,(2)9,(3)12,(4)11,(5)19,(6)36,(7)141,(8)304 .....

Code: <9,0,3,1>

Sequence: (1)3,(2)9,(3)86,(4)26,(5)12,(6)126,(7)65,(8)210 .....

.....  
 .....  
 .....

Note that, when the software lists the sequences in the output file, it also automatically specifies, before each sequence, the unique-code of the sequence itself.

Considering the previous example, we can quickly choose an item to be used in the IQ test, using the second sequence of the list:

- item: 3,9,12,???,19,36,141
- solution: blank item = 11; history-code = [9,8,2,8]

**Observation.** Again, we underline how we can immediately verify that the series adjacent to the one we have chosen differs from the latter in at least one element (discarding the elements which occupy the same position as the blank item); since the OS are sorted alphabetically, this provides us with the certainty of the uniqueness of the solution.

### 4.3 Main flow

The main flow of the program includes three principal blocks:

- receive (and check) the input sequences;
- perform the Derivation Process;
- print the output (the OS).

The core of the algorithm is the “**Derivation Process**”.

This process consists of 1, 2 or 3 steps of derivation.

For each step, the program uses a “pair of sequences”.

In each pair:

- The first sequence is the series to which the step of derivation must be applied. It is a “primary” series (that is, an IS), if it is used during the first step of derivation; otherwise, it is a “derived” sequence (that is, it is the output of the previous step of derivation).
- The second sequence represents the “rule” which will be applied to the first sequence of the same pair. The second sequence of a pair is always an IS.

In the program, the second sequence of a pair is considered as an “array of positions”, because it contains the positions that the algorithm uses to search, within the first sequence of the same pair, the values to be inserted into the output sequence generated in the current step of derivation.

According to this pattern, the first sequence of a pair represents an “array of values”: a subset of these values will constitute the output sequence of the step of derivation.

So, for each step of derivation, the software uses two sequences: the first one is denoted by “seqVal”, because it contains the “values” used by the algorithm of derivation, and the second one is denoted by “seqPos”, because it is an array of “positions”.

A third sequence involved in the step of derivation is the output sequence, which is denoted by “outSeq”.

All of the sequences are managed through arrays of integer. The whole set of the IS and the whole set of the OS are represented as matrices (two-dimensional arrays) of integers.

During a step of derivation, we run through the seqPos, as follows: for each element of this sequence, the software inserts into the seqOut an integer equal to the element of seqVal which occupies (within the seqVal) the position number “I”, where “I” represents the value of the current element of the seqPos.

If, during the cycle, we get a value in the seqPos which is higher than the cardinality (total number of elements different from -1, not including the fields of the “unique code”) of the seqVal, the software adds a fictitious element “-1” to the outSeq and the cycle ends. This fictitious element represents an “impossible value”, because we do not manage negative integers, so it only serves to indicate that the cardinality of the seqVal has been overtaken during the creation of the outSeq.

Once the cycle is completed, the software updates the unique-code of the series.

Obviously, the main flow of the program performs all the steps of derivations which can be applied to the set of IS; the total number of OS is equal to the sum of 3 terms:

- $11^2$  : total number of OS which can be obtained through a single step of derivation;
- $11^3$  : total number of OS which can be obtained through 2 steps of derivation;
- $11^4$  : total number of OS which can be obtained through 3 steps of derivation.

Therefore, the global number of OS is equal to  $11^2 + 11^3 + 11^4 = \mathbf{16093}$ .

## 4.4 Technical data

### Programming language:

Java (jdk 6.0).

### Hardware requirements:

- CPU: Intel or AMD dual core processor or higher;
- Memory: at least 2 GB of RAM;
- Operating System: any version of Windows or Linux (Java software is platform independent).

### Elapsed time (total time spent for a complete elaboration on a system with the minimum hardware requirements):

Between 90 and 120 seconds.

## 5 Operative rules (guidelines for the psychometrist who oversees the test)

The test session starts with the presentation/explanation of the test itself. It is essential that each candidate receives the same information, which must be provided in the same way by the psychometrist.

The guidelines that each psychometrist should respect are as follows:

- 0- The test rules explanation has to take a minimum of 30 minutes and a maximum of 45, plus 15 minutes for answering candidate's questions (if any). He should not respond about the rules that he has clearly explained and he cannot give other details, except the ones which are included in this list.
- 1- At the beginning of the "explanation time", before starting to talk about the test, the psychometrist must deliver a pamphlet to each candidate. He must also hand to each candidate a protocol sheet, a pencil with a rubber on the top and a pen. The final answers must be written on the answer sheet using the pen, while the pencil can be used by the candidate during the "thinking process", together with the rubber and the calculation paper. No pocket calculators can be brought into the test room. The candidate is not allowed to

introduce further material in the test room, that is, in addition to what will be given to him by the psychometrist: a pencil, a rubber, a pen, a protocol sheet, the informative pamphlet (brochure) and, at the right time, the test itself.

- 2- Inside the test room there should be a clock to let each candidate check the remaining available time before the submission deadline. The psychometrist must notify all candidates about the end of the test.
- 3- The psychometrist should clearly describe the method used to construct/derive one sequence from another one, mentioning an example which uses Fibonacci  $\rightarrow$  Fibonacci sequence and showing the sequence Fibonacci  $\rightarrow$  Fibonacci  $\rightarrow$  Prime-numbers.
- 4- The psychometrist must denote by “IS” the mother sequence and by “OS” the final outputs printed on the exam. He has to refer to “IS” and “OS” during the whole explanation of the test rules.
- 5- The psychometrist has to clarify that for each derivation step, the underlying derivation rule may change, regardless of the rules used in the previous steps of the same item.
- 6- The psychometrist has to clearly underline how a step of derivation manages the cardinality (total number of elements different from “-1”) of the sequence “seqVal” to be derived: if in the “seqPos” (the sequence whose values represent the positions of the elements of seqVal which will constitute the output sequence) we get a value which is higher than the cardinality of seqVal, then we add a fictitious element “-1” to the output sequence and the current step of derivation ends. This fictitious element, as previously noted, represents an “impossible value”, because we do not manage negative integers, so it only serves to indicate that the cardinality of the seqVal has been overtaken during the creation of the outSeq.
- 7- The psychometrist must globally spend 10 minutes out of the total available time (30-45 minutes) to read out the pamphlet in front of the candidates.

## 6 Test rules pamphlet

Before starting to elucidate the test rules, the psychometrist must hand (to each candidate) a two page long pamphlet, containing the following instructions:

- 1- The test contains 35 total items of different difficulty. The items are NOT arranged according to their difficulty (they are randomly shuffled).
- 2- The maximum amount of steps of derivation is equal to 3. Therefore, if you find an item solution using more than 3 derivation steps, it means that your attempt to answer that item is definitely wrong.
- 3- The item subsets are as follows: 3 items  $\rightarrow$  1 derivation; 20 items  $\rightarrow$  2 derivations (5 using the same rule plus 15 using two distinct rules); 12 items  $\rightarrow$  3 derivations (7 using two different rules plus 5 based on three different rules).

- 4- Any term belonging to the sequences and used for the derivations is smaller than  $10^{10}$  (from 1 to 9 digit numbers).
- 5- Each sequence contains an unlimited quantity of terms and each OS contains from 7 up to 70 elements (including the blank term “???”).
- 6- 6 out of 11 total sequences are strictly not “non decreasing” (each term is not greater than the next one), while 5 sequences are free (not recursive positive integer sequences which do not respect the rule  $n(i) \geq n(i-1)$ ).
- 7- Each OS contains the minimum quantity of terms which promulgate a unique solution (only one possible step-to-step pattern being involved), provided that the amount of the terms belonging to the item itself is at least equal to 7. Otherwise, the item has to list the first 7 terms of the final sequence. The only exception is represented by solving patterns which involve more than 500 terms.
- 7b (reminder)- Every blank term (identified with three consecutive question marks) is the result of only one retro-analytical reasoning pattern involving, at most, 500 terms for each derivation step.
- 8- In order to help the candidate during the solving process, (but only with aspects which are in no way related to any kind of intellectual ability) each term of the IS is preceded by a progressive number put between brackets. The latter represents the position of the given term inside the IS it belongs to. For example, taking as IS the Fibonacci’s sequence, we have: (1)1,(2)1,(3)2,(4)3,(5)5,(6)8,(7)13,(8)21,(9)34,(10)55,(11)89,(12)144,(13)233,...
- 9- The answer provided by the candidate must contain the ??? value (the unknown element) and a sequence of numbers which describes the pattern used to produce the given item. For example, using 3 derivation steps, a possible solution (for a given item) could be: “**???**=**58**; [**10,5,3,5**]”. If any of these four boxes are left blank, or contain wrong values, the item score is zero. If an item is not based on 3 derivation steps, the candidate can leave blank the final box/boxes of the array or put inside one “X” for each blank box (e.g. “**???**=**198** ; [**6,2,X,X**]” or simply “**???**=**198** ; [**6,2, , ]**”).
- 10- There are no penalties (i.e. negative scores) for wrong answers, so trying to guess is an advantage for the candidate. However, the criterion used in order to discriminate among candidates who achieve the same raw score takes into account the number of answers provided by each participant: the fewer answers a candidate provides, the higher his final rank will be (inside the given *raw score group*).

## 7.1 Norming process

After the implementation, we are ready for the operative testing phase. We require at least 200 ascertained gifted candidates (IQ 130+,  $\sigma=15$  on some standardized tests). It is not important if we administer one specific test or a group of equivalent tests with the same parameter settings: the norm will be unique and this psychometric tool will be ready to be used for screening highly gifted children, searching for the mathematically talented [5], future chess grandmasters or excellent scientists.

Following the approach previously introduced, it will be theoretically possible to create a kind of extremely/profoundly gifted young children database as well. The ranking, obtained through the performance achieved on this innovative generation of tests for the high range, would be able to open up new frontiers, letting us offer many new opportunities.

It would be optimal to link these achievements to some prizes and/or other awards from academic institutions or cultural organizations. They would be assured that their future students are (at least) “highly gifted”, without any doubt about possible cheating.

Alternatively, the test could be used for ranking purposes [1], referring to the grants explained above. With reference to particular fields such as mathematics (number theory, group theory, etc.), cryptanalysis, geometry and professional chess, the selection method based on the raw score achieved on one test of this kind would be a more reliable talent indicator, especially if compared to an equivalent performance on the GMAT test or the GRE.

## 7.2 A faster option for screening candidates

Premised on the fact that no norm, by construction, can ever be stable over time (due to the Flynn effect, a general mean fluctuation related to various population means, etc.) [6-12] and that any IQ related performance is affected by many external factors, there are other interesting options which can be explored in order to reach our goal. To facilitate a reduction in wasted time, with the aim of obtaining a cheaper operational procedure, it will be possible to use a battery of two different performance tests on a not-previously-tested population: a collective tool, plus an individual one. In particular, under the assumption that we have a target group of candidates above the  $+3\sigma$  from the expected mean, we could use the Raven’s Matrices [9] setting the cut-off at  $+2\sigma$  from the mean. Candidates who pass the first (preliminary) test would be admitted to the second phase (taking one test belonging to the test family previously described). In fact, Raven’s Matrices-based tests (which are among the highest g-loaded psychometric tools available) are quite similar to the one presented in this paper.

## 8 Conclusion

At this time, there is no test for assessing high IQ that can be entirely immune from the risk of cheating: both supervised and high range tests are prone to this problem. On the other hand, a lot of high IQ people ask for a tool which is as truthful and reliable as possible. In particular, gifted students’ screening is very important in order to invest in the future from a meritocratic perspective, gaining individual richness from the evaluation of youths’ talents. By sustaining individual capabilities we contribute to psychological good health, but there is more: it is a real strategic resource, directed towards social development and human progress. Investing in talent will bring benefits not only to the economy, innovation and employment, but will also help social cohesion, progress and competitiveness, promoting the growth of the *Knowledge Society*.

For this purpose, we have developed one method which opens the door to a new generation of IQ tests for the high range and which is able to protect any candidate from the risk of cheating by others. The first advantage, compared with “normal” standardized tests, is that, once it has been explained, it is entirely automatized, and the test itself is different from candidate to candidate, changing from one examination session to another one as well. Nevertheless, each version maintains exactly the same difficulty as the others, even if the chosen “IQ target” we are interested in can be changed (in a given range) by simply modifying some of the parameters listed in section 3.

“Classical” numerical tests are the ones most commonly affected by cheating (it is very simple to exchange information on a given item via the internet), while our family is absolutely secure. Our tests are numerical, only involving integer sequences, but they are mainly focused on pattern recognition and fast reasoning/deep analysis: they are similar to the best g-loaded tests, such as Raven’s Matrices. For this reason, it is possible to adopt the RPM as a preliminary test to make an early screening of the candidates, putting the IQ cut-off at  $+2\sigma$  from the mean.

Unlike other supervised tests, the ENNDT (Equally Normed Numerical Derivation Tests) are specifically designed to investigate very high IQs (above the  $3\sigma$  level).

In theory, a person can engage in them more than once, because his performance is not vitiated by the training effect facilitated by multiple attempts. This could also help monitor the development of very high IQs with increasing age.

## Appendix

The derivation process illustrated, when applied to sequences constructed by the decimal expansion of an irrational number, always produces, after a few iterations at the most, a very small quantity of sequences which repeat cyclically.

In detail, we can analyze what happens considering three of the most famous irrationals: the number of Euler (Napier), pi and  $\sqrt{2}$  [10].

Considering the decimal expansion of “e”, we have that (for  $n \in \mathbb{N}_0$ ), after  $2n+2$  iterations (steps), the recurring string is as follows: 2,7,1,8,2,8,1,8,2,8,8,1,1,8,1,2,7,1,7,8,2,8,7,8,7,1,7,1,2,8,8,2,8,1,7,7,...

In fact, the cycles of the terms of the original sequence ([A002193](#) of the OEIS) are:

1⇒2⇒1⇒...  
 2⇒7⇒2⇒...  
 3⇒1⇒2⇒1⇒...  
 4⇒8⇒8⇒...  
 5⇒2⇒7⇒2⇒...  
 6⇒8⇒8⇒...  
 7⇒1⇒7⇒...  
 8⇒8⇒...  
 9⇒2⇒7⇒2⇒...  
 0⇒//

**N.B.**

Remember that the terms which are “zero” are removed (by covenant) from the first step of the derivation process.

Referring once more to the decimal expansion of “e”, we have that (for  $n \in \mathbb{N}_0$ ), after  $2n+1$  derivations (steps), and the recurring string is similar to the sequence [A119506](#) of the OEIS (the only dissimilarity is represented by the different interpretation given to the elements “0” in the original sequence – with the related transformation cycle).

Observing the decimal expansion of “pi”, we have that (for  $n \in \mathbb{N}_0$ ) after  $2n+2$  derivations (steps), the recurring string is: 3,1,4,1,5,5,4,5,5,3,5,5,5,3,5,3,4,3,5,4,5,4,3,3,5,...

In fact, the cycles of the terms of the original sequence ([A000796](#) of the OEIS) are:

1 $\Rightarrow$ 3 $\Rightarrow$ 1 $\Rightarrow$ ...  
 2 $\Rightarrow$ 1 $\Rightarrow$ 4 $\Rightarrow$ 1 $\Rightarrow$ ...  
 3 $\Rightarrow$ 4 $\Rightarrow$ 3 $\Rightarrow$ ...  
 4 $\Rightarrow$ 1 $\Rightarrow$ 4 $\Rightarrow$ ...  
 5 $\Rightarrow$ 5 $\Rightarrow$ ...  
 6 $\Rightarrow$ 9 $\Rightarrow$ 5 $\Rightarrow$ 5 $\Rightarrow$ ...  
 7 $\Rightarrow$ 2 $\Rightarrow$ 3 $\Rightarrow$ 4 $\Rightarrow$ 3  
 8 $\Rightarrow$ 6 $\Rightarrow$ 5 $\Rightarrow$ 5 $\Rightarrow$ ...  
 9 $\Rightarrow$ 5 $\Rightarrow$ 5 $\Rightarrow$ ...  
 0 $\Rightarrow$ //

Again, with reference to the decimal expansion of “pi”, it follows that (for  $n \in \mathbb{N}_0$ ), after  $2n+3$  successive derivations, the (unique) recurring string is as follows:

4,3,1,3,5,5,1,5,5,4,5,5,5,4,5,4,1,4,5,1,5,1,4,4,5,...

(while, after the first step, we have 4,3,1,3,5,5,1,9,5,4,5,6,5,2,5,4,1,4,6,1,9,1,9,1,4,4,6,...).

Finally, analyzing the decimal development of  $\sqrt{2}$ , it follows that (for  $n \in \mathbb{N}_0$ ) after  $n+2$  derivations (steps), the (unique) recurring string is:

1,4,1,4,4,1,1,4,1,4,1,1,1,1,4,4,4,4,1,1,4,4,1,4,4,4,1,1,1,4,1,4,4,1,1,1,1,...

(while, after the first step, we have 1,4,1,4,4,1,1,2,1,4,1,3,1,6,2,4,5,5,1,1,5,5,3,4,4,4,6,1,6,5,3,5,5,9,6,9,3,...).

Thus, the cycles of the terms of the original sequence ([A002193](#) of the OEIS) are:

1 $\Rightarrow$ 1 $\Rightarrow$ ...  
 2 $\Rightarrow$ 4 $\Rightarrow$ 4 $\Rightarrow$ ...  
 3 $\Rightarrow$ 1 $\Rightarrow$ 1 $\Rightarrow$ ...  
 4 $\Rightarrow$ 4 $\Rightarrow$ ...  
 5 $\Rightarrow$ 2 $\Rightarrow$ 4 $\Rightarrow$ 4 $\Rightarrow$ ...  
 6 $\Rightarrow$ 1 $\Rightarrow$ 1 $\Rightarrow$ ...  
 7 $\Rightarrow$ 3 $\Rightarrow$ 1 $\Rightarrow$ 1 $\Rightarrow$ ...  
 8 $\Rightarrow$ 5 $\Rightarrow$ 4 $\Rightarrow$ 4 $\Rightarrow$ ...  
 9 $\Rightarrow$ 6 $\Rightarrow$ 1 $\Rightarrow$ 1 $\Rightarrow$ ...  
 0 $\Rightarrow$ //

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